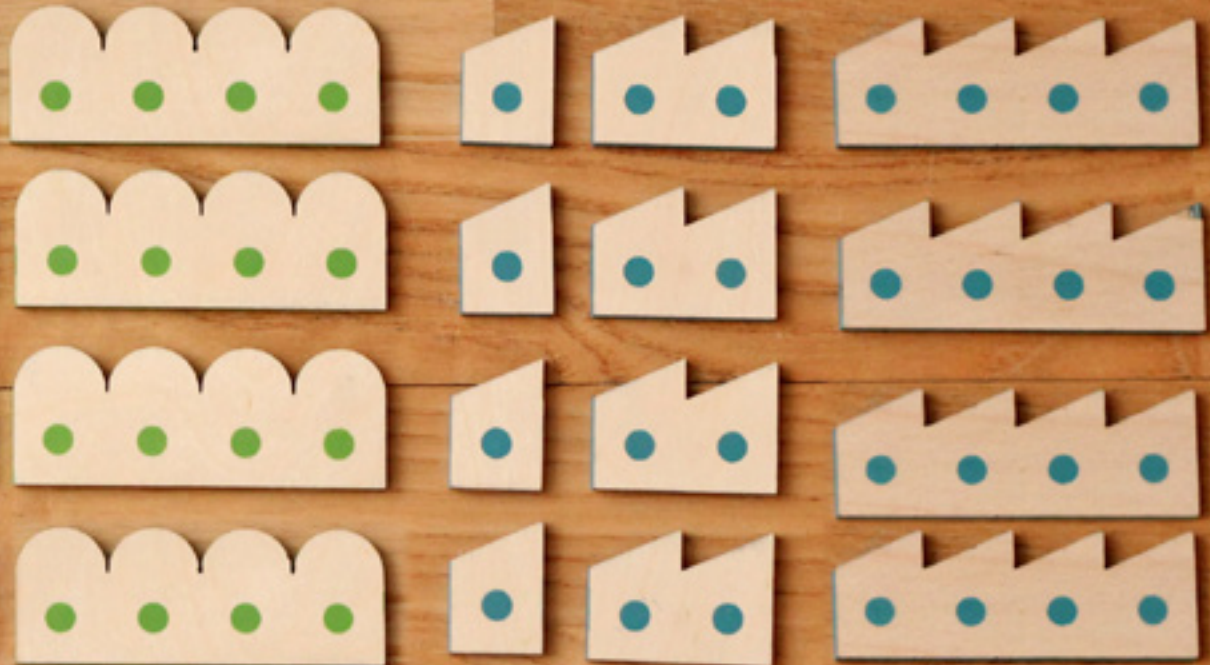




play,
music
& math

Designing
playful learning
experiences

Mariano Velamazán
MfA Interaction Design
Umeå Institute of Design
2016



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Abstract

Superbleeper is an electronic music instrument that is played using math concepts. It invites 3-6 year old children to play with the math they have to understand according to the Swedish curriculum. This math foundation for the youngest kids is about measurement, shape, patterns, time, change, quantity, set and order.

The tests carried out with children in different contexts show that electronic music can be a way to embody and enjoy the use of math concepts in a creative way.

A personal note

When I was a kid, my math teacher used to say that math was beautiful and that it was everywhere but I couldn't see it. I enjoy learning anything but math was always difficult for me; why?

This project is, in a sense, about making visible the invisible; about revealing what is hidden inviting people to explore and understand. It offers people the possibility to make connections between different things. It is trying to make people ask themselves what is going on, what is going to happen next and what would happen if "I tried this".

Introduction

How do we want our kids to learn? Exploring the world or exploring a screen? What is the role of technology in learning? Companies and governments are going through a process of digitalization, how should that process be in a learning environment? This project tries to provoke thought and discussion around those questions.

We all worry when we hear that performance of Swedish (and Spanish) students shows indications of decline (1). Education systems, students and teachers struggle with this reality.

Even if this decline didn't exist, I believe technology can help to improve learning experiences. That is enough motivation in itself to try to explore the possibilities in that area.

Finally, it is about the role of design in this context. It is a vision of design as a vehicle for making people think and be active and creative using their things.

Initial research

Fortunately, I found a collaboration partner that was working in the same problems I was interested. The Interactive Institute Swedish was just starting a project called Pedagogical Interactive Math Visualizations (2)

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1 A report from PISA 2012 (Programme for International Student Assessment) showed a stark decline in the performance of 15-year-old Swedish students in all three core subjects (reading, mathematics and science) during the last decade to below OECD averages.

and I was invited to participate in a workshop to start thinking about the topic. The theme of the workshop was to start with a lesson chosen by one of the invited teachers and try to improve it. I had the role of facilitator and started asking the other participants (a school administrator, a computer scientist and the teacher) what was what they thought people found difficult when studying math.

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First workshop organized for the Interactive Institute PIMvis project.
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2. "The aim of this project is to improve learning and engagement of 12 to 14-year-old kids (årskurs 7-9) using a collaborative design process involving students, teachers, pedagogs, interaction designers and programmers. The project will create a series of demos organized into a "Visions Catalog" to demonstrate how interactive visualisation can support different directions for digitisation in mathematics. [...] The design of several ... demos are intended to be used as digital artefacts to provoke thought and discussion from all collaborators throughout the participatory design process."



For those who struggle with math, there are many factors that make this learning difficult. Most of those factors

seem to be under the general idea of math being abstract and disconnected from the real world. More precisely, these were the outcome themes of that first workshop:

- Language barrier: word problems (stating a math problem in words) aim to connect math concepts with real world situations but many students do not understand the statement, the phrasing of the problem.
- Choosing topic: students are given problems and exercises that are not interesting to them. If they could choose the topic, they would be more motivated.
- Visualization/physicality: many students have a hard time understanding math concepts because they find them abstract. If they could visualize/ touch/sense them it would make their life easier.
- Cross-subject activities: sometimes the problem is that students don't see the application and connection of the math they learn with their real life and interests. The situation could improve if the activities linked different fields.

Choosing a combination of some of the problem areas described above, I thought then, this project would explore the boundaries between cognitive science, math education and interaction design.

The ground

But that was just a first contact with the topic. Out of personal interest I had read a lot about learning and about math and music. All that was unorganized and unclear in my head so I made sure the core concepts were clear. The following is a selection of those concepts that ended up being relevant for the project.

Some people may not know what constructivism is and some others will not know what a mathematical set is. These concepts are presented just in order to establish a well defined common ground for the project from among all the different possible perspectives. They are presented here very briefly and only in order to show some theoretical foundations that have affected the decisions taken through the project. They go from a very general idea of learning to a quite very narrow description of the math and musical concepts on which the project builds.

Teaching vs. learning

One of the first sources of misconception when working and explaining the project was if I was trying to teach anything, what was it? I have some experience as a teacher but I have always felt uncomfortable assuming that a person -even less an object- could teach anything. I always prefer the word learning and my interest is in designing learning experiences. Why?

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3. According to Wikipedia “teaching is the imparting of knowledge by a teacher or other knowledgeable person”. A teacher is a person who provides “learning, or the acquisition of knowledge, skills, values, beliefs, and habits.”

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4. On the other hand, “learning is the act of acquiring new, or modifying and reinforcing, existing knowledge, behaviors, skills, values, or preferences and may involve synthesizing different types of information.”

It is my understanding that the first term implies an idea of learning as transfer of knowledge. The second is an activity that implies an active learner, not a passive receiver of knowledge from a knowledgeable person.

The purpose of this project has been to put an object in the world that may facilitate a learning experience. In that sense, the goal was to provide an experience that may provoke learning through play and exploration.

But learning is still a very broad concept, a more concrete view of learning is still needed to understand the ground on which the project has been built.

Embodied cognition

Is learning a just-mental process? Was I going to design another tablet app to practice adding and subtracting this time with musical notes?

The term embodied cognition describes the body as an active agent while learning. Consequently, not only the brain is responsible for our cognition processes. Movement, perception (visual, aural, tactile, etc.) and the context are the main aspects of the body that shape human cognition [13].

The fact that the body and the senses are an important part of learning has been central to the project. For many people, learning is just an intellectual process. In this project, the physicality of the tokens, the act of stacking them together and then the composition of the sequence are an example that the body, hands, eyes, ears and brain are all involved at the same time. It is my belief that this way the experience of learning and playing is richer and more engaging.

Having established that learning involves the mind and the body doesn't explain how we learn. I will try to cover briefly my view of that process in the next section.

Constructivism

Not so many years ago there was the belief that knowledge could be transferred from one person to another through a clear explanation. Most of our education systems are organized around that assumption.

New visions on mathematics education (situated learning) have contested that idea and most of the new visions on learning take constructivism as a starting point. What is the core idea of constructivism?

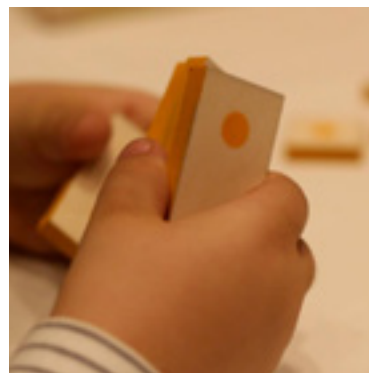
Building on ideas from John Dewey, Maria Montessori, Jean Piaget and Lev Vygotsky among others, constructivism is a kind of philosophical framework that explains how people learn building on experiences, creative ideas and prior knowledge [14].

But how can we learn mathematics based on experience if math is so abstract that there is nothing mathematical that can actually be perceived? Let's try to explain now how we understand people construct mathematical knowledge out of their experiences and bodies.

Metaphors and embodied mathematics

Saunders Mac Lane (the inventor, with **Samuel Eilenberg**, of **category theory**) proposes that mathematical concepts are ultimately grounded in ordinary human activities, mostly interactions with the physical world [15].

Lakoff and Núñez [5] explain that from a very basic innate



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5. Constructivists claim that students do not simply learn by being told, and that all students should receive opportunities to construct and recontextualise knowledge from meaningful learning experiences (Lerman, 1996; Brandsford, Brown, & Cocking, 1999). Constructivist theories differed from the behaviourist theories offered at the turn of the twentieth century, but like their predecessors they represented knowledge as something that is constructed within people's heads. More recently situated theories of learning (Lave, 1988; Greeno & MMAP, 1998) have offered a new perspective on the development and use of knowledge that pertains in interesting ways to the provision of opportunities for mathematical modelling. Situated theories have taken the focus off individuals, suggesting that knowledge emerges as a series of interactions between people and the world. [21]

ability to count, add and subtract small numbers, people are able to construct complex mathematical structures using metaphors. The core and most basic mathematical metaphors are grounded on human experiences like collecting, constructing, measuring and moving.

For example, one way we understand the mathematical idea of set is as a container (with an inside, an outside and an element that acts as border or frontier between the two).

Some basic metaphors described by those authors used in my project are:

- Arithmetic is object collection/construction
- Change is motion
- Sets are containers, objects
- Numbers are sets, object collections, physical segments, points on a line
- Geometric figures are objects in space

Now let's jump to present the elements of the project that are going to be mixed together: math and music through free play.

About math

For many of us, mathematics is about arithmetic operations (adding, subtracting, multiplying, dividing...), algebraic equations, geometry and calculus. That is what we were taught at school. If people don't see numbers, symbols and equations they don't see math.

But the way mathematicians think of their discipline and work is not so layered and not so operation oriented.

Many mathematicians describe their work as the study of patterns, structure, space, quantity and change [16].

In accordance to the previous definition, the Swedish curriculum for preschools describes the following goal:

“develop their understanding of space, shapes, location and direction, and the basic properties of sets, quantity, order and number concepts, also for measurement, time and change” [17]

Maybe the mathematical concept of set and the way order is treated in our project need to be further explained.

A set is a collection of objects. These objects (also known as the elements or members of a set) can be anything [18]

A sequence is an ordered set [19]. Unlike a set, order matters, and the same elements can appear several times.

This project has tried to concentrate on providing an experience for developing an understanding of shape, set, quantity, order, measurement, time and change through tangible technology and having music as the output of those concepts and the relationships between them.

But, what is the connection between all this and electronic music? And more important, why electronic music anyway?

About electronic music

Choosing electronic music is motivated because it is the sound of technology, machines, electricity and computers. As such, it is the sound of our time. Computation technology has translated a vast amount of phenomena to num-



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6. Computers have given composers new opportunities to automate the compositional process.

bers. Electronic music has its origins in synthesizers, which were operated following numerical parameters from the very start. Music was one of the first creative disciplines to use technology as a new instrument or media with its own expressive qualities. Music can also be understood as embodied math [10]. As an sensory, emotional way to give math an output or at least to make visible the math behind it (order, structures, proportions, etc.).

The project also grows on musical set theory. A musical set is very similar to a mathematical set. The members of a musical set are notes or pitch classes (a numerical representation of a note) [20].

Another source of inspiration is algorithmic composition which is a formal procedure to make music with mathematically predefined sets of rules that normally include the introduction of some degree of randomness.

Finally, a few words about the third component; the one that closes the perfect circle.

About play

Children get absorbed when they play. The possibility of setting your own rules and goals and get transposed into a state of flow has always fascinated me. It is that kind of drive that makes people spend hours and hours learning to play an instrument or becoming better at a video game.

According to Johan Huizinga [9], play is a free activity in which the constraints of reality are suspended. It is an experience that absorbs the player and creates its own boundaries of space and time.

There are several play patterns but one that is relevant to the project is:

Object play, such as playing with toys, banging pots and pans, handling physical things in ways that use curiosity.

This project seeks to open up the curiosity of children by mixing the physical and the abstract, the emotional and the rational, the expression and the reflection.

Related projects

Finally, we present other sources of initial inspiration from related projects.

Chess

Chess is a very old game that is also played using math concepts: the geometry of the board and the movements and the logical thinking that it requires make it a perfect reference for this project.

Tangram

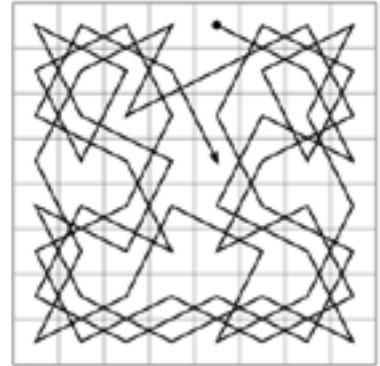
Another ancient game that uses a set of wooden blocks to play with mathematical concepts and shapes.

Lego

The idea of system implied in Lego blocks needs to be mentioned in this section. Lego pieces can also be seen as sets and they are also designed following mathematical proportions in a modular way.

Music blocks

According to their own description, “Toddlers compose music by arranging--and rearranging--colorful blocks



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Mathematicians [Euler](#), [Legendre](#), [de Moivre](#), and [Vandermonde](#) studied the [knight's tour](#).

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Created by Ilmari Karonen - Created by Ilmari Karonen using chesstour-svg.pl based on Image:Knights tour.png by Gdr., Public Domain, <https://commons.wikimedia.org/w/index.php?curid=1479164>

in the electronic toy's base. The included cartridge allows each of the musical blocks to "play" a musical phrase of a Mozart melody. Placing a block onto the base on its different sides produces the sounds of different instruments playing the same musical phrase"

This project was interesting because it is also a sequencer but used in a much less creative way and very little math is involved. It is also difficult to understand the decision to put Mozart's music on it.



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Zizzle Zoundz music toy

Zizzle Zoundz

"Zoundz™ creates a fusion of self-composed music with an accompanying light show. With it, users can create musical light shows never seen before. They can create their own riffs by placing one of Zoundz™' pawns on an interactive "hot spot" on the sound board. Each pawn associates with the sound it makes and by placing them on different locations and in different combinations"

Zoundz™ is a much more creative toy since it let's the child record her sounds and play them but the rest is completely set and there is actually, like before, very little math and music playing/learning.

The opportunity

Maria Montessori wrote in *The Montessori Method* [22]:

“It would be possible to have an idea of the form of the quadrilateral without knowing how to count to four, and, therefore, without appreciating the number of sides and angles. The sides and the angles are abstractions which in themselves do not exist; that which does exist is this piece of wood of a determined form.”

If I want to facilitate math learning/playing experiences, I think that the real problem for children is to connect an abstract mathematical concept with their real life experience. Mathematics is a very abstract domain; we can't have real life experience of the idealized mathematical objects. For example, a circle is a mental abstraction that cannot be perceived through the senses [12].

I believe that the most meaningful learning is embodied (either situated in context or through play), so these were questions that I started to ask myself:

- can we develop systems enhanced by technology (digital and/or physical) that facilitate embodied mathematics learning experiences for children?
- can we make these experiences playful?
- if we learn through metaphors (5), can we create



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By MattThePuppetGuy at English Wikipedia - Transferred from en.wikipedia to Commons by OverlordQ using CommonsHelper., Public Domain, <https://commons.wikimedia.org/w/index.php?curid=8929032>

systems that become a ground on which children can create their metaphors?

Research question

More precisely, the aim of this design research project was established like this:

Create a product that allows 3-5 year old children explore mathematical concepts such as set, quantity, shape, measurement, time, order and change. Using tangible technology it will try to provide an embodied experience of those math concepts through music and play.

It was time to start defining what things I had in mind I had to do and wished I could do.

Goals

Probably the first general goal was that children had fun and really wanted to play it and create music. The component of self-motivation was key.

More importantly, the math shouldn't be about operations; it should be about being creative with math. As a consequence, the math behind the toy/instrument/tool had to be precise but also implicit. No symbols and no formal training was sought. It had to be an embodied experience of mathematical concepts. Playing and exploring with these concepts (sets and other math properties) would have basic electronic music as the output. The interactions and representations of set theory would be coherent (but not literal or obviously represented; they would be embedded in the shape, interactions and the function of the object).

There would be a mapping, a precise correspondence, between (the selected part of) math and (the selected part of) music. Ideally, this mapping would become a coherent system that could grow for older children and for more complex math and music.

It would be a standalone object that would not need neither a computer nor any other requisite.

Wishes

That the object was part of a system that could grow was a secret dream that was there from the very beginning.

In that sense, it would become a kit of modular components that could be combined to add more functionality. For example, there would be a module for visualizing on a screen (when connected to a tablet). There would be a module for guided tasks and activities (when connected to the internet). There would be a module for programming music. There would be a module for recording and sampling the recorded sounds. Finally, there would be a module for creating live jam sessions between several children.

Results: the exploration process

If I wanted to make visible the invisible, create an embodied learning experience and invite children to explore and play with a new toy I had to understand children, how they learn and what is it that makes them play, have fun and be involved and engaged in an activity. It was my feeling that they are not asked about how they want to learn. I think user-centered design is very powerful but in learning, the user is the learner and s/he is, I think, normally forgotten. I also needed to make things for them to test and play with. I also wanted to observe them in order to learn from them and use my observations to improve my prototype. The purpose of the prototype was to make people think and talk about the possibilities of playing with math concepts in order to create beats.

The first part of the project was about gathering information from books, papers and, as I said before, from users.

The readings that guided my process in general were two: *The Role of Children in the Design of New Technology* [8] by Allison Druin and *Design Research Through Practice* [23]. I will stop to talk about what I learned from them.

The Role of Children in the Design of New Technology

The paper by Allison Druin was a revelation to me, exactly what I needed to read at that moment. She describes four roles children can take during the design process: users, testers, informants and design partners. Each of them increases the responsibility of the child in the decisions taken. I would have loved to have children as design partners but I think the role of kids in my process was basically as informants during this phase and as users and testers later on in the process. As users, they are provided with products that they try out. Their feedback is used to improve the product. As testers, the role is similar but the product is not finished, it is a pre-release or a prototype; which was my case during the refinement phase. When they are informants they are asked to participate in different moments of the process starting from sketches. This is very much what I did with them during ideation workshops described later on.

Constructive Design Research

The authors refer to constructive design research as a process in which construction -in my case prototypes and mockups- “takes center place and becomes the key means in constructing knowledge”.

They describe four methods among which the two that I think I have been mixing are what they call lab and field. Lab is based on experimentation. The designer/researcher makes things and objects to see how people react while holding some chosen variables constant.

But since the focus is on children learning and playing, it was important to bring also the field point of view. I didn't do my experiments in my lab, I didn't ask children to

come to my desk. I went to their homes and schools to let them play with my prototypes; the natural settings where my design would be used to see what happened there, in the world.

As the authors say “Researchers follow what happens to design in that context. They are interested in how people and communities understand things around designs, make sense of them, talk about them, and live with them.”

Observation

I also have to stop to say a few words about my experience working and observing children. Watching them exploring detail, looking at every hole, listening to every sound even if it was just random noise was thrilling. The insights I got from their behaviour, reactions movements and gestures were just unique. The surprises I got from their answers, opinions and ideas were priceless.

This has been already explained and analysed by others [24, 25] but now I understand better what they mean.

Literature Circle

With the members of the Interactive Institute that were part of the PIM-vis project, I held one weekly meeting around a few research papers concerning math education, visualization, pedagogy, gamification and prototyping. My [conclusions can be read here](#).



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Joyce Chow works for Interactive Institute Swedish in Norrköping. Her feedback was very valuable.



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Invitation to the first interview with students

Students workshops

I contacted twelve students (13-15 years old) in Minerva Skolan. Three one-hour workshops with four of them in each activity were run with the following themes:

- Likes and dislikes out of school
- Likes and dislikes inside school
- Likes and dislikes about music and math

I kept refining the workshops since it was hard to get the information I wanted from the students. At the beginning they were not very open and I only had the time of one lecture: 50-55 minutes. It also needs to be said that part of the workshops were held in Spanish which is the third language of the students. It was difficult to access students and the only way to convince teachers was to use Spanish as the language of the workshop so that they could practice and get something out of it.. In this sense the conditions were far from ideal. Speaking in Spanish made everything very slow and neither fluent nor natural. Nevertheless, in my opinion, the output was still very useful.

I was very interested in two aspects:

- Motivation
- Cross subject activities

All the workshops asked, in general, about what things they liked and enjoyed. From that starting point I tried to concentrate on the things they enjoyed that required effort and discipline. I was trying to find a common pattern for motivation.

All workshops started with one icebreaker activity. After introducing ourselves I delivered an A3 week calendar with green and red post-its. The task was to chose the

three most enjoyed and three most hated activities of the week. Then they had to grade and rank those activities and explain why some of them were more exciting than the others.

Workshop 1

More in detail the first workshop was about activities outside school. These students were the youngest and they didn't reflect much. When they had to compare and explain they just articulated answers like "it is more fun" or "it is more boring". I also had a first version of the activity that was a bit too complex and I spent too much time explaining what they had to do.

In general, their answers confirmed what I expected but some were surprising: reading fantasy books and quite a lot of activities that require effort like playing music, dancing, painting, etc.

Workshop 2

It was held right after the second but with older students and I simplified the activity. When I asked them why they tried something hard but disdained other things that required effort this is what some of them said:

"I am good at it. It can get harder and harder but I want to keep being good at it"

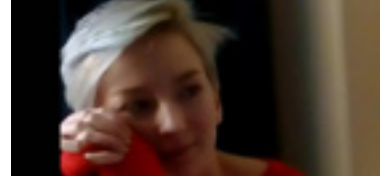
"You can see you get better. In school you get someone to tell you if you are good or not"

"You only see your improvement in a grade"

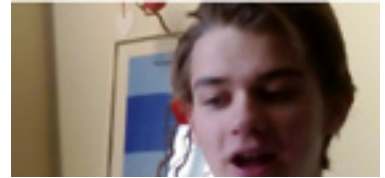
"In math you are forced to think in a certain way, you can't think of a creative solution (I didn't ask about math)"



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First interview with students in Minerva Skolan



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One of the students of the third interview in Minerva Skolan



.....
Second interview with students in Minerva Skolan



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More students in the last interview in Minerva Skolan

This made me realize that if I let them talk more, and elaborate on their answers, they would give me very valuable information. I just had to make them feel comfortable and make them forget that I was there.

Workshop 3

In this last workshop I concentrated on their most loved and hated subjects in school. After they ranked and graded them, I asked them specifically about their thoughts and feelings about math and music.

The outcome from them can be summarized with these two ideas: the importance of the teacher and the feeling you are good at something as the main factors for motivation. After being asked in more detail they reflected on the fact that math forces you to think in a certain way and that there is only one correct answer; there is no point on it “because someone has already figured it out for you”. It was also eye-opening to listen to one of the kids say that learning was like getting a kick when you get to use all you know and have been practising in order to solve something you couldn't have solved before studying and training.

Math educator interviews

They were asked about what are the main difficulties that students find studying math. This was the main answer:

- the concept of variable. Many students who don't (later on) understand the function concept lack in their understanding of what a variable is.
- randomness. This is a known difficult area also.
- the idea of multiple representations. That the same mathematical entity can be described using



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Tomas Berqvist from the Department of
math Education of Umeå University

a graph, a table, a function, a story, or something else. Switching between different representations can be hard for many students.

Electronic musician interview

Elias was very helpful showing me different models of commercial sequencers and describing what is essential and different about them. I also got feedback about some features of the first idea.

Other users interviews

Math and music teachers and experts were also interviewed during this phase but their contribution to the project was more about explaining to me math and music concepts that were not clear and giving me feedback about what students enjoy and find difficult in their classes.

Textbooks and official curricula for math and music



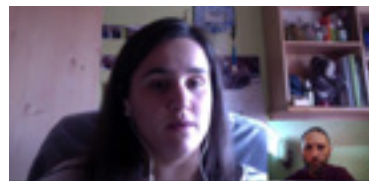
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Fragment of a music text book showing the proportions between notes



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Elias Warg, electronic musician



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Johan von Ahn (music teacher)

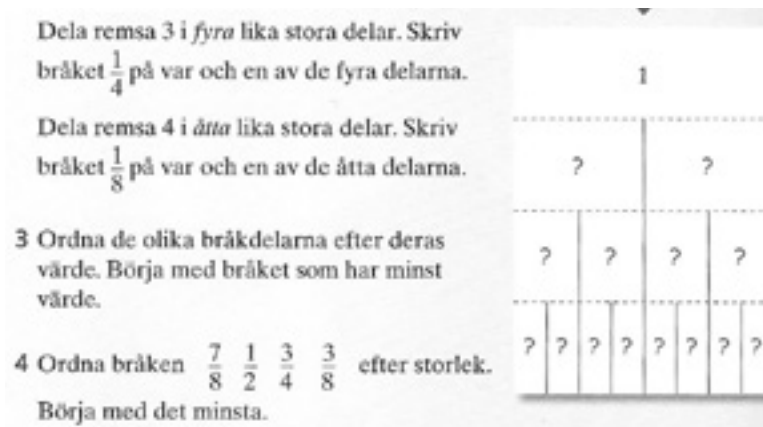


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Helena Cuesta (music engineer)



.....
Stephan Lindberg (sound designer)

.....
 Fragment of a math text book showing
 an intuitive idea of proportions



In order to have a better understanding of the requirements of the education system at the target group age, the curriculum of math and music was analysed.

From the curriculum for Swedish preschools I got this goal:

“develop their understanding of space, shapes, location and direction, and the basic properties of sets, quantity, order and number concepts, also for measurement, time and change.”

Textbooks were also reviewed searching for inspiration. Above all I wanted to find content that was common to both disciplines but that was studied separated. In the pictures below, mathematical fractions and the proportions between the notes are part of the same concept. But they are explained in two completely different ways without relating to each other.

The main outcome of this research activity was to focus on:

- set theory
- ratios and proportions

Research conclusion: design and pedagogical principles

As a general conclusion of this first weeks I came to define a few points that would be the principles that guided my decisions later.

The pedagogical and design principles were:

- using math concepts (patterns, structure, measurement, time, change, quantity) to create and play rhythms
- using tangibles to visualize and sense these concepts
- the tangibles would have precise mathematical properties
- played with simple movements and gestures that would reinforce the meaning of the concepts
- playful

I also tried to identify the problems that would force me to take important decisions at the end of the research phase:

- Should the target group be changed to younger kids? Should it be tried and tested with different ages?
- Offer opportunity to reflect
- Make it more social and collaborative

I decided to keep working with kids in order to take decisions based on my observations and activities with them.

So, in order to put something on the table to start the

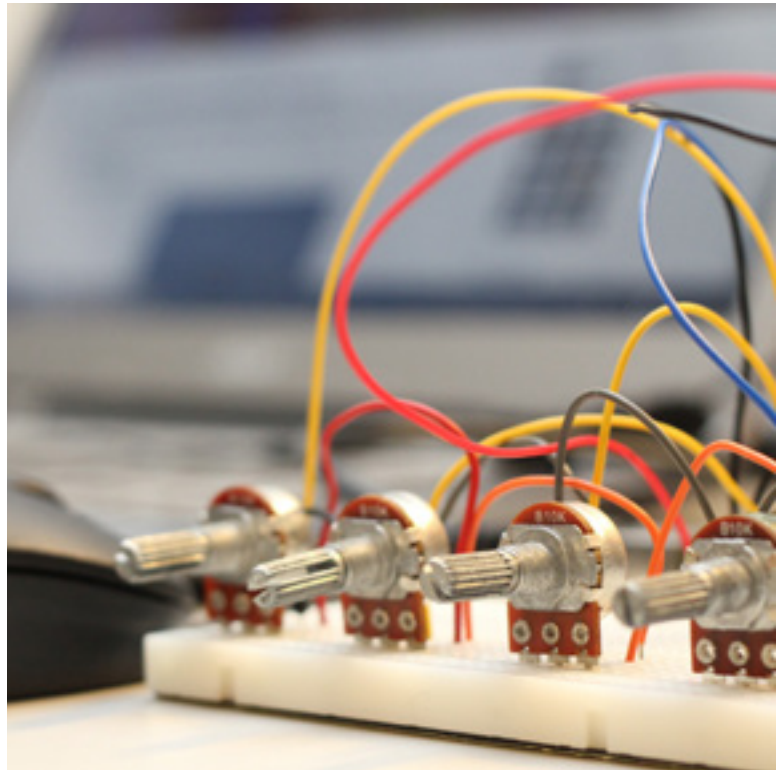


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The first of a series of paper prototypes. It was a way of showing children prototypes that “looked good and serious” but also easy and fast to make.

conversation, a first paper mockup was created and shown to kids, one designer and two teachers. From these first interviews it was clear that the product resembled too much to the existing electronic instruments and that there was very little math involved.

Ideation

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This first sketch prototype played rhythms created with potentiometers. It was the result of a long search for a good sound synthesizer.



Ideation phase started testing –through Wizard of Oz– a first paper mock up with four kids and two teachers

(math and music). The teachers comments were very positive but the kids didn't understand the point of the experiment. It seemed that the kids didn't accept that the prototype was just an experience of what it could be a later finished product. They didn't think it was "serious" to tell them that "when you do that, this would happen". In general, it was not what you call a success.

With this in mind, I set myself to learn to run and create productive workshops with kids as design partners/ informants, ideate with them and make sure that what I was doing meant something to them. I followed an approach of building and improving on previous workshops to keep refining them.

Workshop 1

A new workshop was defined to explore more possibilities. The goals of this second workshop were:

- Get inspiration from how people represent sound with geometrical shapes
- See them talking to each other and working together in order to represent a sound or a rhythm
- See them interpreting visual / physical and geometrical material as sound

Wooden blocks, cardboard cut outs, paper, scissors, pens and markers were the materials provided.

Conclusion & observations

The best was that the students were engaged and motivated: they said -and it seemed that- they enjoyed the workshop.

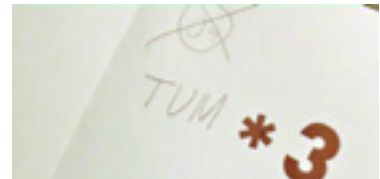
All of them were part of a kind of "extra math time" group so maybe that explains why they were so engaged.



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Wizard of Oz with students in Grubbe Skola



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One of the results of the first ideation workshop with kids from Minerva Skolan



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One of the results of the first ideation workshop with kids from Minerva Skolan

Numbers and operators were used –almost always– as a parallel representation with little mathematical precision. The numbers and operators were not used in a real mathematical way. It was a mixture of visual approximation and “decoration” that made the thing look very mathematical but not very precise. Nevertheless, some of them were very clear to understand.

I will describe an example of what happened when using tangibles and numbers to represent sound. One of the kids had a quite complex representation of the sounds that also used more than 15 numbers. As a proof that he was completely involved in the activity (maybe because it involved all his senses and brain) he realized that one of the numbers that he had placed in his sequence was missing. And he also knew what number it was: he said: “What happened to my four?”

They used triangles for edgy sounds and curves for soft sounds.

Workshops 2 and 3

After that workshop, I wanted to focus more on ideating a possible toy or instrument and test it to position the target group towards a younger age. I did that by defining and running two workshops with design students from UID.

From these two new workshops I got the impression that all of them focused on including the math as arithmetic operations. I wondered if I could give a more creative view of math. One that is more about finding and describing patterns. I also got the impression that the target group should be younger and that the product should be more an autonomous sound machine kind of toy.



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One of the results of the first ideation workshop with UID students

Workshop 4

A new workshop was designed to test if those impressions were correct. The goal and purpose of the workshop -with music students- was to focus more on the music and creativity side. Design students saw math as arithmetic operations, math and science students saw music as just emotional feedback. I wanted to see how music students saw math and ask them to create a machine that would create music using math concepts. Their view of math was about calculating what notes would “sound good” (following rules of chords and harmony). I also spent some time with them building a cardboard prototype so that all the functions had to be seen and explained.



One of the results of the last ideation workshop with kids from Minerva Skolan

Concept development

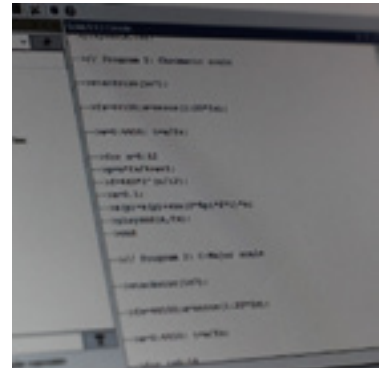
After every workshop, I spent some time reflecting on the outcomes of them, the vision of the students and looking for other sources of inspiration.

One of them, that helped me start thinking about how to treat sound in a mathematical way, was to see how Matlab and Scilab worked with waves and how they could be used to synthesize sounds and to be the output of mathematical functions.

After some sketches a system emerged for representing music notes in a physical-tangible way. These physical tangibles were designed following precise mathematical properties. They could be combined following mathematical concepts.

Video prototypes

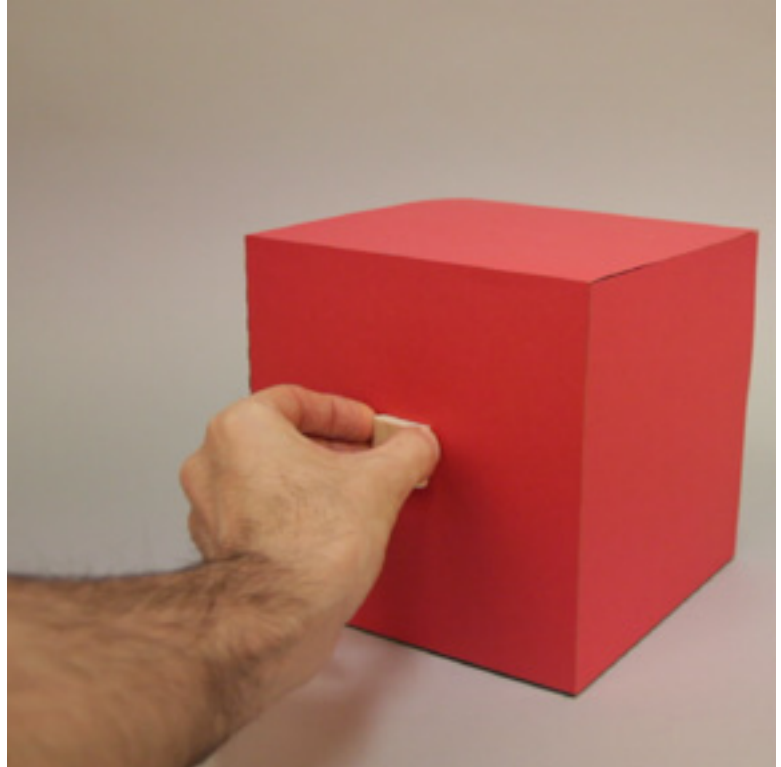
After choosing two of the most successful ideas, more refined paper mockups were created and iterated. Finally,



Scilab screen showing the commands to synthesize sound from math functions

two quick models were built and draft concept videos of the chosen ideas were the outcome of this phase.

.....
Video frame from the first video prototype of what was called then the “knock, knock” box. The actual functionality of the knocking was one of the first features to be left out due to similarities with existing projects.



The “knock, knock box” would invite children explore shapes, measurement, time, order, change, quantities and notes and rhythms.

The “knock-knock box” was a cube on which sides sound tokens could be placed and a sequence of sounds would be created. Lifting the box would make all the elements of the sequence to play higher pitches. Lowering the box

would provoke the opposite. Knocking on the box would make it “answer” with the same number of knocks as the user in a sort of echo.

Both ideas rely on a common ground. The system of blocks that represent the different sounds, scales and waveshapes was implied in both of them. Both ideas were also based on playing those sounds in an ordered set; a sequence.

The main difference between them was the target groups addressed. In the “box” version, the audience was 3-5 year old while in the other idea, the audience was 6-9.

Both videos were shared with many of the users that had been part of the research and ideation. Feedback from collaboration partners and teachers was considered and the “knock, knock box” idea was chosen because:

- It was more important to develop properly the system of representing sound in a physical and mathematical way
- The “table” idea would have been impossible to prototype in a way that children were able to test it and gaining experience working with kids was a core principle of the project.



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Paper prototype previously built to test the “Superbleeper table” idea.

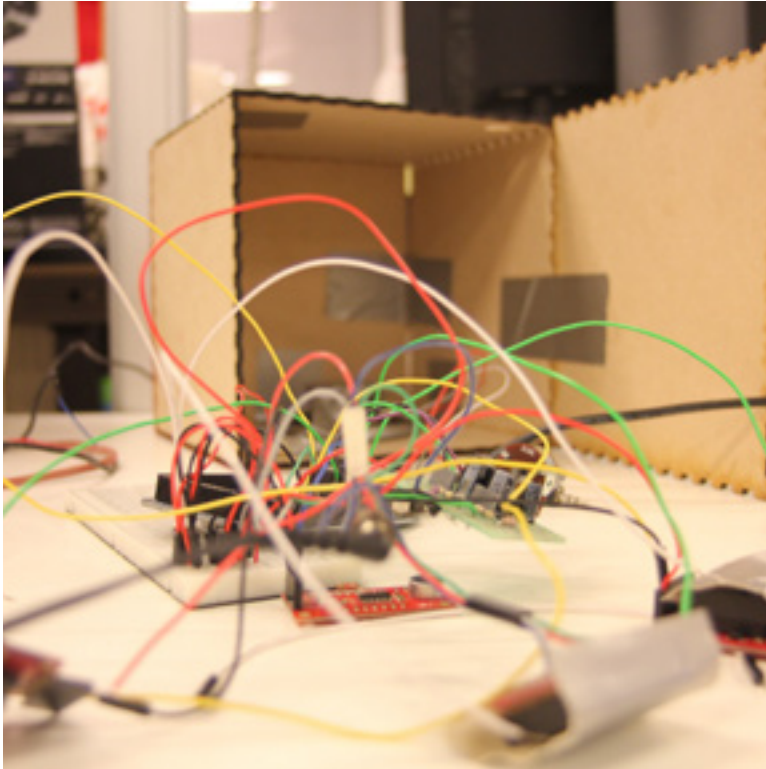
.....
Video frame from the first video prototype of what was called then the “superbleeper” table. This was the idea that was planned to develop.



The “table” idea would let children play with operations and compose more complex compositions thanks to the use of variables. It would be possible to add, subtract, multiply and divide sounds. Besides, children would be able to change the tempo and the pitch of the sounds multiplying the beats per second module or the Hz module. More complex compositions could be created storing sequences inside variables and performing set operations with these variables.

Prototyping microinteractions

The very first days of prototyping were focused on testing and choosing the core functionality.



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First prototype of the box. I started with the common functionality of both previous ideas in order to postpone the decision on which one to choose.

Another dimension of the project was the microinteractions contained in the video prototype of the chosen idea. I realised, thanks to Niklas Andersson, I had to pay attention and refine the structure of the moments in which children introduced changes to the system. In the first part of prototyping not much attention was paid to these details because I was concentrated on putting up

together the technology. But they became the core of the iterations and refinement process described later.

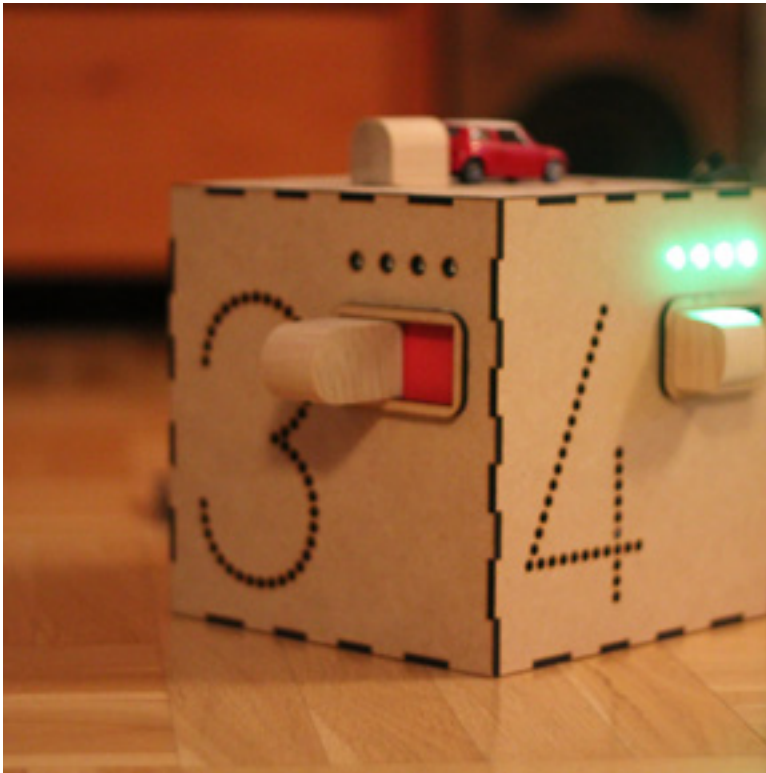
According to Dan Saffer (11), microinteractions “are contained product moments that revolve around a single use case—they have one main task. [...] What makes effective microinteractions is not only their contained size, but also their form. A beautifully-crafted microinteraction pays attention to all four parts of a microinteraction [...] A Trigger initiates a microinteraction. The Rules determine what happens, while Feedback lets people know what’s happening. Loops and Modes determine the meta-rules of the microinteraction.”

From the video prototype I defined some core microinteractions:

- Turning the box on and off
- Adding sounds to the sequence
- Changing pitch of the whole sequence

Through the iterations two more microinteractions were added:

- Defining the duration and pitch of a note token
- Changing tempo of the whole sequence
- Changing the volume of one note token



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First working prototype of the box. The idea was to test the functionality, the feedback, the blocks... A first impression.

Clapping & counting

The idea of counting with sounds that was part of the original video prototype was quickly implemented in two different ways. In the first one, the box would trigger a series of sounds to which children would have to answer -clapping or knocking- with the same amount of sounds. The problem with this version was to filter out other sounds that were not part of the clapping of children. In the second iteration, an adult would trigger -through a remote control- a number of sounds and the children would answer, clapping or knocking, with the same



amount of sounds. The problem with this idea was that it was against the spirit of the project to depend on an adult to be able to play. The purpose was to offer an explorative object that children would have to discover on their own motivation and interest.

The functionality of clapping and counting was left out because it was not oriented to creativity and it became boring after a short while. This was quickly tested with my own child.



Shape and size of blocks

Trying to give a tangible sense of number and quantity was another important aspect of the prototype at this phase. Different shapes for the blocks were explored to find the best combination of size, texture, material and communication of number sense.



How to position the blocks on the sides of the cube was also investigated. From quite a wide area to facilitate the position to a small circular orifice that forced children to precisely place the token inside it. Quick tests were again performed with my own child.

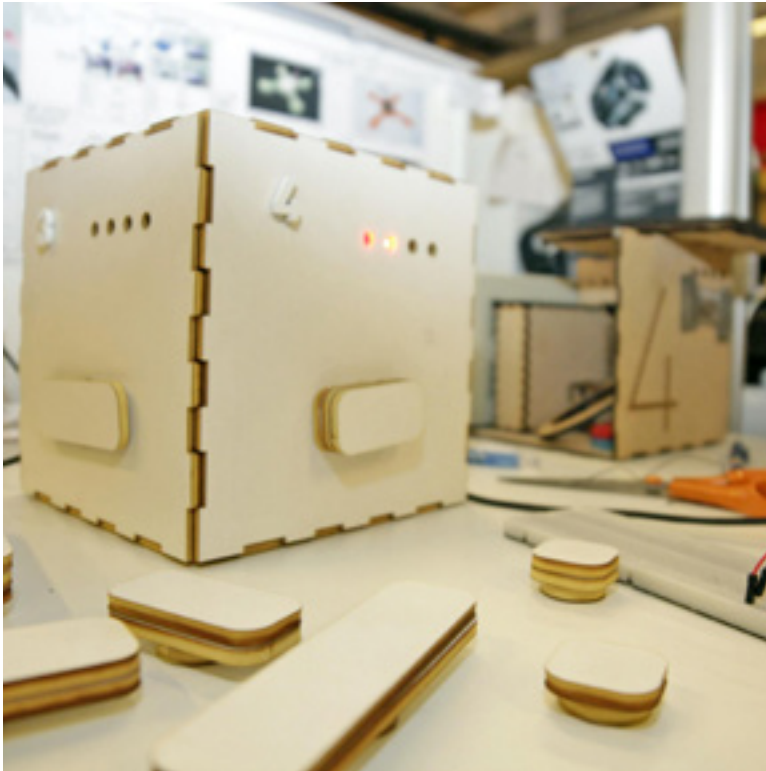
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Different interactions on the shape of the blocks

Feedback elements

Sound in the form of the musical notes that the blocks represented was the most important feedback but in order to make the mathematical properties of the blocks and the sound more clear four lights were added to each side of the cube to make clearer the relationship between the block and the numerical value of the duration of the note played.

Description of iteration 1



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First iteration on the first working prototype. Numbers have been reduced and shape and placement of blocks have been changed.

It has to be noted that until this point of the project, the blocks were made out of one piece of wood. They were not the outcome of children piling different small blocks. They were proportionally cut as one piece blocks. They already varied in width and height depending on the duration and pitch of the sound they represented but they were not made out of layers of wood stacked by the children. It was now that the blocks became glued layers

of wood to give the impression of having been stacked on top of each other. They were made out of foamcore board.

To make clear the idea of order implied in the sequence (one of the sides is defined as the first side) little numbers were placed on the corners. These numbers were already a simplification and reduction from previous numbers that were found too big considering their function was only to indicate the cardinality of the side in the sequence.

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First real user test of the first iteration. Children didn't understand anything and were quite confused.



This version of the box was tested with three kids in Rymdem Förskola. The purpose of the test was to check:

- If children understood the connection between width of the blocks, the lights and the duration of the sounds played
- If they could find the relationship between the height of the blocks and the pitch of the sounds played
- If the children found it playful and fun

The test was carried out in the presence of a Swedish educator because I can't speak the language. The activity was very open. Since none of them knew anything about the project I let the children play freely and explore the toy for a few minutes and then asked the teacher to join and try to figure out what was the point of it.

Outcome of iteration 1

Neither the children or the teacher understood any of the first two points described above. But they found it fun and intriguing.

The numbers on the sides and the lights at the same level made them think that it was a kind of puzzle in which they had to match the number of lights lit up with the number of each side.

The blocks were made out of white foamcore board and the layers were not very visible.

The prototype didn't work very well when three children played with it at the same time which was very frustrating for them because the behaviour was not very consistent.

The core system and concept

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The first version of the complete concept. The system of layered blocks and light feedback is already settled.



The layers of height of the blocks were to be stacked by children so that they could explore the consequences of their design and changes. They would also be colored and therefore more visible and became a sign to be interpreted.

The different colors would obey to the different scales of the different sets of sounds.

The numbers on the sides were completely removed.

More lights were added to indicate the duration and the height of the block placed.

The group of lights was moved to the center of the side and closer to the input port for blocks in order to make clearer the connection between the lights and the properties of the blocks placed on each side.

The volume was improved.

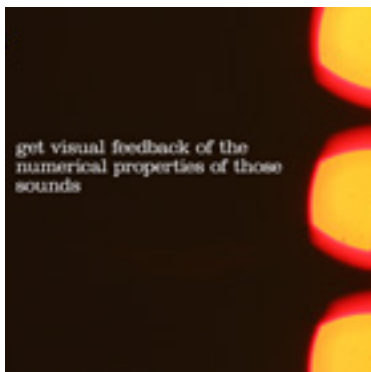
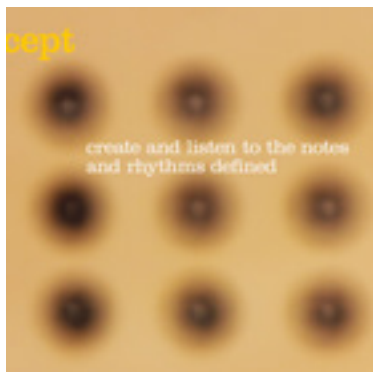
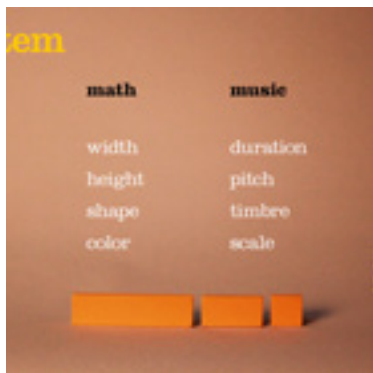
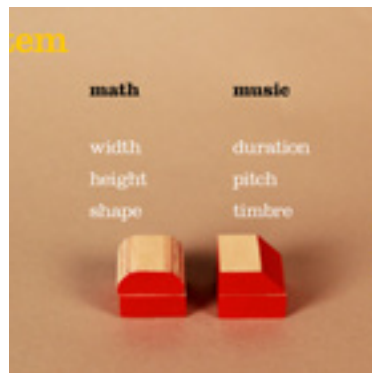
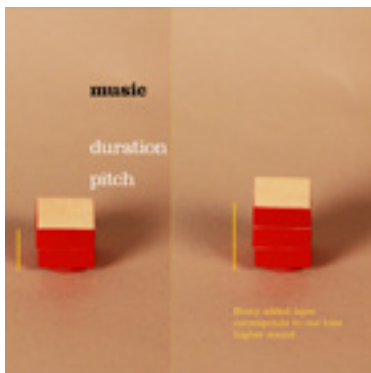
The material was changed to plywood to make it more solid.

Tilting the box would make the tempo increase or decrease depending on the direction of the tilting.

As shown in the video prototype, lifting or lowering the box would make the set of sounds of the sequence to play higher or lower frequencies.

And finally, with these changes, the core functionality, appearance and behaviour of the concept was defined.

Results: the exploration process



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The first description of the system.
Fragments of the slides of the process gateway presentation

Second iteration: testing & evaluation



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First user test of the second iteration.
The blocks were not clear but children
(and parents) understood the concept

With the changes described above and the addition of batteries (to remove the USB power cable the prototype was tested again with two 5 year old children in an office environment (neither their preschool, nor their home but their father's office). The father was present.

In this case, the father was quite active asking questions to the kids from the very beginning and trying to get answers from them. The father didn't care so much about

the music side and went straight to the mathematical possibilities. One of the children was not very interested in that and liked the music and the lights but the other followed the questions of the father and very soon started to be able to predict how many lights each block would lit up.

When I asked the child was able to explain how would be the sound although in terms of duration. He didn't sense the change in pitch but I hadn't had the time to color the layers of blocks and make them higher as I planned after the first iteration.

The third iteration



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In this iteration, the wooden blocks were refined

In this test, in Hedlunda Förskola, the teacher was present to help me translate. She is in charge of the music and dancing activities with children. We let the children play freely and after a while started asking questions.

The children said that one block would lit up many lights and play a long sound when they had a long block. They could also predict that there would be more lights if the block was higher than another one with the same width

but less layers. They counted the lights on without being asked although were not able to predict precisely how many would lit up.

They couldn't describe what happened to the sound when higher blocks were placed (higher pitches would be played) but they certainly appreciated it was a beat box since the girl said that if I repaired all the bugs I would have "robot music machine".

The blocks were colored red and lights were the same color. The lights when the box was empty were green. When the children placed the first block a 1x1 block, they heard a short low sound and a red light and they thought they had made something wrong, teacher included.

After around 30 minutes, when they realized it was a beat box they started to loose interest and went to play something else.

Limitations of my tests

The tests were very valuable but there were also serious limitations.

To begin with, the tests were quite short. There was not an elaborated test during a long period of time in an specific context.

There was also a language barrier since I could not speak Swedish. Since children don't communicate their ideas in a reflective way, my conclusions are based on observation of the kids.

Concerning the concept, the children did not test the

stacking of the blocks due to limitations of the technology of the prototype. The RFID readers cannot read more than one RFID tag at a time so the children were given prestacked blocks of notes for them to insert and test in the box.

But the worst problem was the bugs of the prototype that prevented children from a consistent behaviour of the sequencer. Since the input ports were a combination of a RFID reader and a magnetic sensor, if the children did not place the blocks quickly inside the input ports, the RFID reader would read the tag but the magnetic sensor would not read the block as inserted and the sequencer would not play the corresponding sound. This resulted in a very frustrating experience for many kids that stopped playing a few unsuccessful tries.

Refinement of microinteractions

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Final version of the design waiting for children to play with it



As a result of the last iteration some changes were included.

When the sequence was empty (no blocks on the sides of the cube) the lights would be white. If blocks were added to the sides, the lights would light up in the color of the block (its scale) but red would be avoided. A number of lights would lit up according to the width and height of

the block placed on that side. The sound corresponding to the mathematical properties of that block would be played.

The lights were visible thanks to circular holes that were made to the wood of the cube. These holes were covered with white acrylic. This way it was clearer that the lights were output information (feedback) and that they had nothing to do with the input port (the circle orifice where the blocks are inserted) that triggered the interaction.

The only input port was actually a orifice where tokens are inserted improving the affordance of the box.

New sound tokens were prototyped so that they were smaller and lighter since if they were too big and heavy, they would fall off the input ports. Children didn't understand how they could be combined to create different types of sounds, different waveshapes. New shapes, more explicit and not interchangeable were designed to make the microinteraction of defining a note even clearer.

The sounds of the different shapes were actually implemented in the box. Sine, square and triangle waveshapes could be played by the box. The musicality and quality of the sounds was also improved using [an existing synthesizer](#).

Final design

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The system of blocks is the most important refinement in the final design



In order to improve the feedback of the microinteractions some more details were refined.

Feedback on the top side

One light close to each side of the top side of the cube was added to indicate what side of the cube was playing at any given moment. This was trying to solve the problem that

only two sides of the cube are visible from any point of view so there was a lack of sense of the whole sequence.

A sense of time and change

In order to provide a better communication of the duration of the notes, the lights light up one column at a time at proportional intervals. Not all of them at the same time as before. This animation of lights also reinforce the sense of sequence since they give a sense of movement and direction that is also helpful to show that the sides are sequential and that there is an order between them.

The colors of the lights also correspond to the colors of the token (the scale).

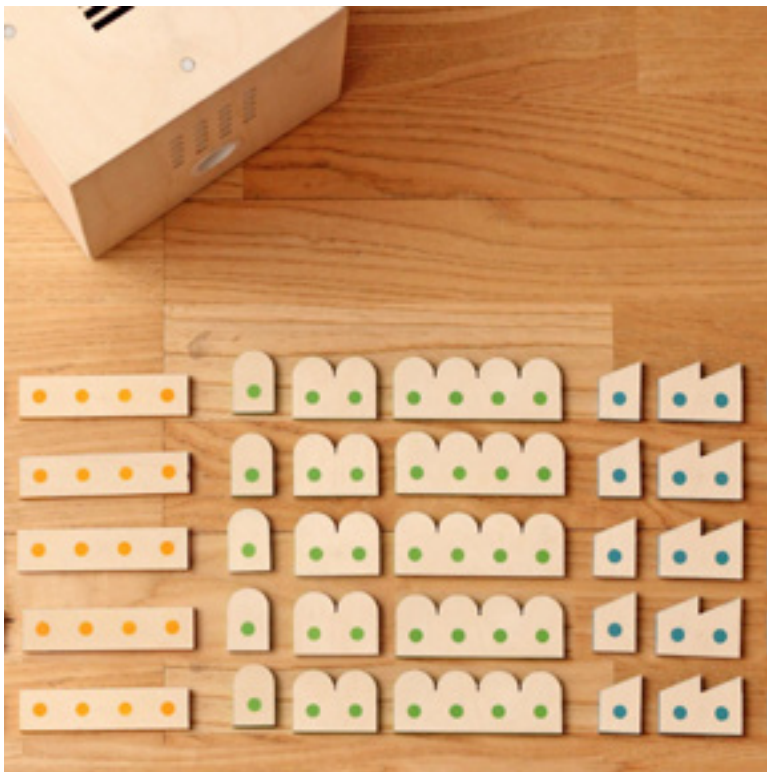
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The light in the input port when there are no blocks inserted is an action trigger



A trigger for the first interaction

When the sequence is empty there is a pulsating light in the input ports to show clearly to children that is a hotspot for interaction. When the sequence contains one member, the lights that indicate time start lighting up so that children realize the four sides are part of a whole and are ordered.



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The circles were added to the blocks in order to reinforce the concept of quantity and the experience of number sense. Also to facilitate the connection with the lights

More number sense

Dots have been added to the tokens to give them a visual representation of the quantity of units of time they represent. It is also easier to relate to the lights they light up.

More to explore



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The user tests improved the design and the experience of play

Through all the user involvement along the whole process it has always been very encouraging to realize that the

concept is full of potential for further exploration of possibilities to experiment and improve.

A brief description of the most interesting ones found so far is given in the following sections

Activities and tasks

For those who prefer a more guided, less free exploration of the product a guide showing possible activities concerning math and music could be very inspiring. This guide could include activities about sets, combinatorics, arithmetic operations and time.

The system grows for older kids

For older kids, more complex math could be explored. See the video prototype for a rough inspiration of the possibilities. Specially the creation of more complex compositions based on the concept of variable.

Playing together and playing live

To allow interaction between children more than one box can play at the same time. It would be very interesting to offer the possibility of synchronising the nearby boxes so that the same sides played at the same time. This way more complex rhythm combinations could be explored by children playing together and they would sound more musical as well.

Boxes could be other polygonal prisms based on the triangle, pentagon or hexagon in order to explore the musicality of each geometry and the combinations between them.

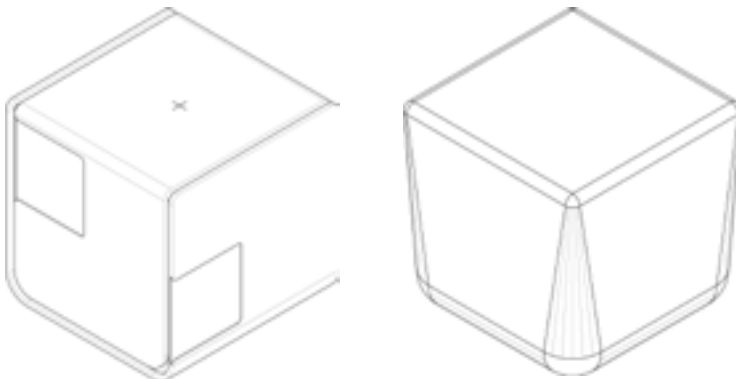
For a more embodied and richer experience extra sounds could be added when tilting the machine left or right. These sounds would depend on the angle of tilting and the distance to a surface like the floor. This would also give a feeling of live performance and improvisation.

The shape

The prototype looks intentionally as a prototype. It was my aim to remain in the space of exploration of possibilities. But the shape of it is very determined by the limitations of the technology used. In this sense it doesn't communicate the intention and functions of the object.

With the help of Rodrigo Marin we elaborated a first draft of the main characteristics of the shape of the object. The principles were that it should have four sides, that it should show its structure and assembly in a very transparent way and that it should communicate the possibility of the movements.

First explorations



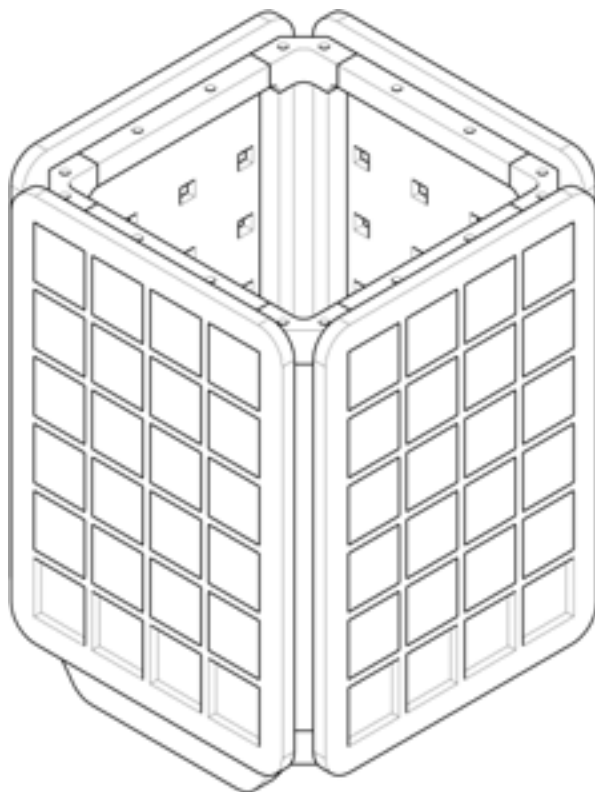
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Exploring asymmetric feedback lights and the affordances for the possible tilting of the box

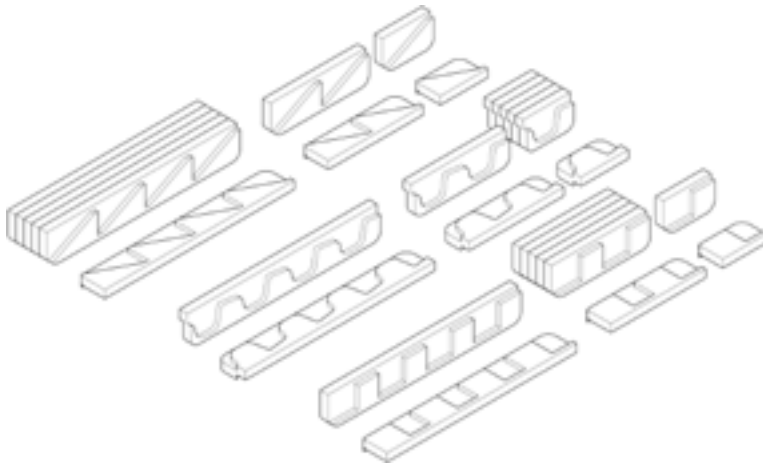
More explorations

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An exploration of the lights filling up all
the side

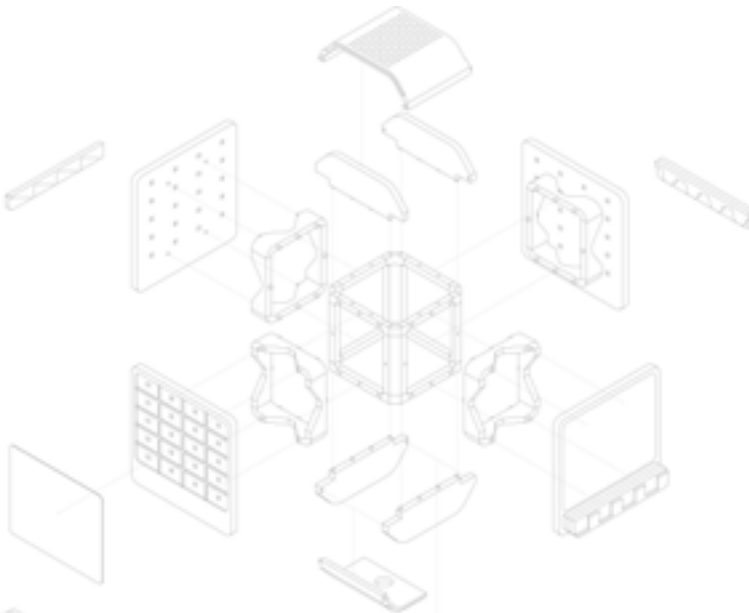


A new system for the blocks



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In this exploration of the blocks, it is not possible to combine blocks from different waveshapes



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Exploded view of the modular interior

The discussion



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Talking to a preschool math educator

I put a new object in the world to make people talk and think about the possibilities of improving the experience of (learning? playing with?) math and music concepts.

When I look back, as part of my own learning process, I have two directions of reflection. One is about learning and the other one is about designing.

Tangibles, Montessori and the digitalization of education

Companies and organizations are going through a deep process of change due to digitalization. How is this process going to affect learning and education? This project tries to raise this question and explore some possibilities.

How do we want our children to grow? Exploring the world or exploring a screen?

Should learning be guided or free?

If we provide an embodied experience of math, how do we go from there to a formal abstraction and learning of that experience?

I honestly do not know the answer to these questions. It is my impression that tangibles and embodied experience in the world are areas that can be further explored when connected to learning. They provide a more complete and engaging process of discovery. I saw it when I did the user tests and when I showed the project to different people.

These questions are the result of this work and the engine of the projects to come.

About prototypes and design that makes you think

During the project, I also had the opportunity to ask myself what design is and what should be.

Too often, in my opinion, design ends up offering the promise of a better life as long as people do what they are told. All the apps that show, suggest, guide, help and assist people in one way or another are an example of this approach.

In this respect, it would have probably been easier for me to teach something telling children what to do. I am not against guiding children and asking them the right question so that they can reflect on what they do and learn. But that alone shouldn't be the purpose of design; just one possible use of the final product, service or process.

People don't seem to learn much when they are being taught. They learn when they can engage in an activity that is meaningful for them.

Through this project I made extensive use of different kinds of sketches and prototypes (hardware, paper, video, etc.) and their power was that they made people think about uses and possibilities of what they were being presented.

It is my understanding that design should create tools to enable people to create and learn exploring the possibilities and uses for those tools.

Acknowledgments

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Thank you Candela, my wife, not only for providing me with all the time I needed, listening to my problems and doubts, giving me feedback through the whole process and even helping me with the leaflet; thank you for being

a source of love and inspiration. Your natural tenderness, taste and talent (and the way you looked at me when we met!) explain why you are the only person I want to impress.

Cooperation with the project/activity

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Appendix 1: the math behind superbleeper

Math = Patterns

From Wikipedia,

“Mathematics (from Greek *μάθημα* *máthēma*, “knowledge, study, learning”) is the study of topics such as quantity (numbers), structure, space, and change [...] Mathematicians seek out patterns”

The sequencer box creates rhythm and light patterns that depend on the mathematical properties of blocks selected by the child.

Sets & sequences

A set is a well defined collection of distinct objects. The objects can be anything: numbers, people, letters of the alphabet, other sets, and so on.

A sequence is an ordered set.

In this project music is understood as a sequence (ordered set) of notes. When children place tokens on the sides of the box they are adding elements to the previously empty sequence. The sequence is played in the same order and

changing the side of the token changes the sequence. The repetition of the sequence in time creates patterns of shapes, sounds and lights.

Notes are treated as a subset of sounds. Single blocks represent an amount of frequency and a duration. When children stack up more than one block they are adding up frequencies to define a higher tone and creating a subset among all the possibilities of sounds available to them: higher or lower pitches, longer or shorter durations and different colors and shapes (that represent scale and waveshapes).

From a musical perspective, the sequencer box of the prototype behaves as a set of notes that are played in order (musical sequence). The notes are defined by children before they are played and that definition is based on mathematical proportions and physical properties of width, height, shape and color. For older children, the system could be extended to become an algorithmic composer.

Other concepts from the curriculum for preschool math

- Combinatorics: there are many possible combinations of blocks and rhythms that can be explored defining systematic activities with the Superbleeper.
- Measurement: the sizes of the blocks
- Time: the different duration of the notes, the tempo of the sequence
- Quantity and number sense: the circles in the blocks, the number of lights the light up
- Change: adding or subtracting blocks to the sequencer box changes the sequence. Changing

just one property of the blocks (size, color, shape, height), changes the feedback.

- Geometry: the different shapes of the blocks have been mathematically designed following geometric shapes that also correspond to the waveshape of the sound they play.

Metaphors

- Arithmetic is object collection/construction; (adding or subtracting while constructing a defined stack of blocks to define a music note)
- Change is motion; (lifting and tilting the sequencer box)
- Sets are containers, objects; (the box is an ordered set of notes, the notes are a subset of all the possible sounds)
- Numbers are sets, object collections, physical segments, points on a line; (the sound blocks are physical segments with points representing duration that can be collected or stacked -creating a subset- in order to define a musical note)
- Geometric figures are objects in space; (the sequencer box is a cube in which every side has a function, the blocks are geometrical forms with proportioned sizes according to duration of the sound they represent. The shapes of the blocks are also geometrical resembling a square, a circle and a triangle).

